

# Line-broadening in gravitational radiation from gamma-ray burst-supernovae

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Core-collapse in massive stars is believed to produce a rapidly spinning black hole surrounded by a compact disk or torus. This forms an energetic MeV-nucleus inside a remnant He-core, powered by black hole-spin energy. The output produces a GRB-supernova, while most of the energy released is in gravitational radiation. The intrinsic gravitational-wave spectrum is determined by multipole mass-moments in the torus. Quadrupole gravitational radiation is radiated at about twice the Keplerian frequency of the torus, which is non-axisymmetric when sufficiently slender, representing a “*black hole-blob*” binary system or a “*blob-blob*” binary bound to the central black hole. We here discuss line-broadening in the observed spectrum due to Lense-Thirring precession, which modulates the orientation of the torus to the line-of-sight. This spectral feature is long-lived, due to weak damping of precessional motion. These events are believed to occur perhaps once per year within a distance of 100Mpc, which provides a candidate source for Advanced LIGO.

LIGO recently completed its second science run [1]. The results demonstrate the feasibility of km-sized laser interferometric gravitational-wave detectors. In the broad band width of 20-1000Hz, advanced LIGO is sensitive to extragalactic events of binary neutron-star coalescence and GRB-supernovae from rotating black holes. This band width also covers emissions from new born neutron stars, binary coalescence of black holes, as well as emissions from rapidly rotating neutron stars. Anticipating the spectrum of gravitational radiation from candidate sources is important in designing optimal detection strategies. While the spectrum of binary inspiral is well-understood to high post-Newtonian order at large binary separations, that of GRB-supernovae is only beginning to be identified.

Core-collapse of massive stars is considered a potential site of gravitational radiation [2, 3, 4, 5, 6, 7, 8]. Here, gravitational radiation is produced by the release of gravitational binding energy during collapse and in accretion processes on a newly formed black hole (e.g, [9]). If accretion is driven by angular momentum loss in magnetic winds, the timescales in core-collapse by magnetic regulated hyperaccretion are relatively short: about 0.05s for the outer accretion disk, and about 0.01s for the inner accretion disk [10]. Hyperaccretion flows are probably strongly turbulent, which would imply a broad gravitational-wave spectrum. Aforementioned studies appear to indicate an energy output, which leaves a range of detectability by ground based detectors of up to about 10Mpc. These events should therefore be considered in the context of core-collapse events independent of the GRB phenomenon, in light of current estimates on the local GRB event-rate of about one per year within a distance of 100Mpc [11, 12]. These studies on gravitational radiation in core-collapse of a massive star do not invoke the spin-energy of a newly formed black hole.

The lifetime of rapid spin of the black hole surrounded by a torus in suspended accretion is about tens of seconds [10, 13]. This represents the timescale of “unseen” dissipation of black hole-spin energy in the event horizon, the rate of which is governed by the poloidal magnetic field-energy in the surrounding torus [10, 13]. This timescale is in good agreement with the durations of long GRBs, which reflect the lifetime of activity of the inner engine [14].

Optimal detection strategies for gravitational radiation from GRBs may be facilitated by a priori knowledge from a specific model. Our model for GRB-supernovae from rotating black holes describes the formation of a GRB accompanied by a radiatively-driven supernova, while most of the energy output is in gravitational radiation [13]. The lowest order quadrupole moment is manifest as a “*black hole-blob*” binary system or a “*blob-blob*” binary bound to the central black hole when the torus is sufficiently slender, giving rise to [13]

$$E_{gw} \simeq 4 \times 10^{53} \text{erg} \left( \frac{\eta}{0.1} \right) \left( \frac{M_H}{7M_\odot} \right), \quad f_{gw} \simeq 500 \text{Hz} \left( \frac{\eta}{0.1} \right) \left( \frac{M_H}{7M_\odot} \right)^{-1} \quad (1)$$

in gravitational radiation. Here,  $\eta = \Omega_T/\Omega_H$  denotes the ratio of the angular velocity  $\Omega_T$  of the torus to the angular velocity  $\Omega_H$  of a rapidly rotating black hole of mass  $M_H$ . This energy output (1) results from a causal spin-connection to the black hole of a surrounding torus with multipole mass-moments. For an intrinsic mass-inhomogeneity  $\delta M_T$  in

the torus, we have a luminosity of gravitational radiation according to

$$L_{gw} = (32/5)(M_H/R)^5(\delta M_T/M_H)^2, \quad (2)$$

where  $\mathcal{M} \simeq M_H(\delta M_T/M_H)^{3/5}$  denotes the chirp mass. The relative mass-inhomogeneity  $\delta M_T/M_H$  is determined self-consistently by the balance equations of energy and angular momentum flux in a suspended accretion state to be about a few promille [13]. The fractions of black hole-spin energy radiated in various channels is independent of the mass of the torus (provided  $M_T$  is a few tenths of a solar mass). The mass of the torus only affects the lifetime of rapid spin of the black hole and hence the duration of the burst.

The energy in gravitational waves (1) is larger than the true energy  $E_\gamma$  in GRB-afterglow emissions [13] (see further [15]), i.e.:

$$E_{gw} \simeq 10^3 E_\gamma. \quad (3)$$

We attribute the GRB-afterglow emissions to baryon poor outflows along an open magnetic flux-tube subtended at a finite opening angle on the event horizon of the black hole. Statistical analysis of GRBs with individually measured redshifts points towards strongly beamed GRB-emissions [11] in the form of a highly anisotropic beam accompanied by very weak GRB-emissions over essentially all angles [12]. This is distinct from purely conical outflows with uniform emissions over a finite opening angle and zero emissions outside. The non-uniform strongly anisotropic beam gives rise to an anti-correlation between inferred opening angle and redshift, given a standard true GRB-energy output in response to a finite flux-limit of a detector. The weak “all-angle” emissions are proposed to be at the level of GRB980425, or about  $10^{-4}$  times the standard GRB luminosities – weak, but significant in allowing nearby GRBs to be identified off-axis as apparent anomalous, low-luminosity events.

A radiatively-driven supernova energy accompanies the GRB and burst in gravitational waves powered by magnetic winds coming off the torus in suspended accretion around the central black hole [13]. The predicted kinetic energy

$$E_{SN} \simeq 2 \times 10^{51} \text{erg} \quad (4)$$

is in good agreement with the observed energy of about  $2 \times 10^{51} \text{erg}$  in the aspherical explosion of SN1998bw [16]. We note that the “hypernova” energies [17] of a few times  $10^{52} \text{erg}$  [18, 19] are isotropic equivalent energies assuming spherical symmetry, not true kinetic energies in the aspherical explosions.

We here study the spectral signature of Lense-Thirring precession in (1). Lense-Thirring precession can be described by nodal precession [20]. In the slow-rotation limit, its angular velocity  $\Omega_{LT}$  [27] agrees essentially to the frame-dragging angular velocity. The orientation of the torus is well-defined, in that misalignment between its angular momentum and angular velocity is of second order,  $O(\theta\Omega_{LT})$ , where  $\theta$  denotes the wobbling angle. This contrasts with rigid body motion, where precession is due to a misalignment between the angular momentum and angular velocity vectors. Lense-Thirring precession is frequently invoked in modelling QPOs produced by accretion disks in X-ray binaries [21]. It represents a general relativistic effect due to frame-dragging, and hence it acts universally on the large-scale structure of a torus surrounding a black hole. A torus which is misaligned with the spin-axis of the black hole precesses with essentially the frame-dragging angular velocity described by the Kerr metric. In Boyer-Lindquist coordinates, we have  $\Omega_{LT} \simeq 2J_H/R^3$  for a black hole angular momentum  $J_H = M^2 \sin \lambda$  in terms of the mass  $M_H$  and the specific angular momentum  $\sin \lambda = a/M$ . Given the angular velocity  $\Omega_H = \tan(\lambda/2)/2M$  of the black hole and the angular velocity  $\Omega_T \simeq M^{1/2}R^{-3/2}$  of the torus, we have

$$\frac{\Omega_{LT}}{\Omega_H} \simeq 2 \times 10^{-2} \left( \frac{\eta}{0.1} \right)^2 \sin^2(\lambda/2). \quad (5)$$

At nominal values  $\eta \sim 0.1$ ,  $\Omega_{LT}$  is about 10% of  $\Omega_T$ , or, equivalently, about 1% of  $\Omega_H$ . The associated precession of the black hole, due to conservation of total angular momentum, can generally be neglected when the angular momentum of the torus is much less than that of the black hole.

Our study considers Lense-Thirring precession in response to an initial misalignment of the torus relative to the spin-axis of the black hole. The active nucleus is believed to form in core-collapse of a massive stars [22] *inside* a remnant envelope (e.g. [13]). The angular momentum of the black hole is commonly attributed to orbital angular momentum, received during spin-up in a common envelope phase with a companion star [17, 23]. A misalignment in the nucleus may hereby reflect misalignment between the spin of the progenitor star and the orbital motion of the companion star. Lense-Thirring precession modulates the orientation of the torus to the line-of-sight, introducing phase-modulation in the line-emissions as observed at the detector. This broadens the emission lines, and may produce side-bands similar to those arising in amplitude modulation. There arises an additional pair of low similar to radiation

coming off a freely precessing neutron star. These low-frequency emissions produce relaxation, and gradually bring the torus into alignment. Their low-luminosities allow the misalignment and its precession to be long-lived.

For the two polarizations of gravitational waves, we have

$$h_+ = 2(1 + \cos^2 \iota) \cos(2\Omega_T t), \quad h_\times = -4 \cos \iota \sin(2\Omega_T t) \quad (6)$$

where  $\iota$  denotes the angle between the angular momentum and the the line-of-sight. Here, we have suppressed a coefficient  $2(e_k/M_H)(M_H/r)$ , where  $e_k$  denotes the kinetic energy of the mass-inhomogeneity  $m$  in the torus, and  $r$  denotes to the distance to the source. Precession of the torus introduces a time-varying angle  $\iota(t)$ , which modulates the strain amplitudes  $h_+$  and  $h_-$  at the observer (Fig. 1). Given a mean angle  $\iota_0$  of the angular momentum of the torus to the line-of-sight and a wobbling angle  $\theta$ , the time-dependent angle  $\iota(t)$  of the same satisfies

$$\cos \iota(t) = \sin \iota_0 \sin \theta \cos(\Omega_{LT} t) + \cos \iota_0 \cos \theta. \quad (7)$$

Substitution of  $\cos \iota(t)$  into (6) produces phase-modulation at the Lense-Thirring frequency. The expressions (6) hereby produce line-broadening by phase-modulation about the carrier frequency  $2\Omega_T$ , according to

$$h_+ = 6 \cos(2\Omega_T t) + \cos(2\Omega_T t + 2\iota) + \cos(2\Omega_T t - 2\iota), \quad (8)$$

and

$$h_\times = -2 [\sin(2\Omega_T t + \iota) + \sin(2\Omega_T t - \iota)]. \quad (9)$$

The resulting wave-forms are illustrated in Fig. 2.

In case of  $\theta \ll \iota_0$ , we may linearize (6)  $h_+ = h_+^{(0)} + \theta h_+^{(1)} + O(\theta^2)$ ,  $h_\times = h_\times^{(0)} + \theta h_\times^{(1)} + O(\theta^2)$ , where  $h_+^{(0)} = h_+(\iota_0)$  and  $h_\times^{(0)} = h_\times(\iota_0)$  refer to the strain-amplitudes (6) with  $\iota = \iota_0$ , and  $h_+^{(1)} = -\sin(2\iota_0) [\cos([2\Omega_T + \Omega_{LT}]t) + \cos([2\Omega_T - \Omega_{LT}]t)]$ , and  $h_\times^{(1)} = 2 \sin(\iota_0) [\sin([2\Omega_T + \Omega_{LT}]t) + \sin([2\Omega_T - \Omega_{LT}]t)]$ . For small wobbling angles, therefore, phase-modulation reduces to amplitude modulation, producing a carrier frequency accompanied by two side-bands at frequencies

$$2\Omega_T - \Omega_{LT}, \quad 2\Omega_T, \quad 2\Omega_T + \Omega_{LT}. \quad (10)$$

The ratio of the amplitude line-strengths of the side bands to that of the carrier satisfies

$$K \simeq \theta \left( \frac{1 + \cos^2 \iota_0}{1 + 6 \cos^2 \iota_0 + \cos^4 \iota_0} \right)^{1/2} \sin \iota_0, \quad (11)$$

where we used  $\Omega_{LT} \ll 2\Omega_T$ . Averaged over all angles  $\iota_0$ , we have  $\bar{K} \simeq \theta/2$ . A wobbling angle of about  $30^\circ$  typically produces side-bands of relative strength 20% (taking together  $h_+$  and  $h_\times$  in each side-band).

Low-frequency emission lines are found by considering the projections of a precessing torus onto the celestial sphere, as shown in Fig. 1. Translating the geometrical picture of [32] to the case of a torus around a black hole, we observe that (1) projections along the spin-axis of the black hole on the celestial sphere produces an ellipse which rotates at twice the Lense-Thirring frequency; (2) projections of the torus along directions in the orbital plane on the celestial sphere produce a line-segment which oscillates at the Lense-Thirring frequency. A precessing torus hereby introduces a spectral anisotropy, whereby emissions at twice the precession frequency are preferentially along the spin-axis of the black hole, and emissions at the precession frequency are preferentially along directions in the equatorial plane.

The strain-amplitude of the low-frequency emissions satisfies [32]

$$\mathbf{h}_{LT} = \sin(2\theta) \frac{\Omega_{LT}^2 \Delta I}{2r} \hat{\mathbf{h}}_{LT}, \quad (12)$$

where

$$\hat{\mathbf{h}}_{LT} = \mathbf{e}_+ \left( h_+^{\parallel} \tan \theta + h_+^{\perp} \right) + \mathbf{e}_\times \left( h_\times^{\parallel} \tan \theta + h_\times^{\perp} \right) \quad (13)$$

and

$$h_+^{\parallel} = 2(1 + \cos^2 \iota) \cos(2\Omega_{LT} t), \quad h_\times^{\parallel} = 4 \cos \iota \sin(2\Omega_{LT} t), \quad (14)$$

$$h_+^{\perp} = \frac{1}{2} \sin(2\iota) \cos(\Omega_{LT} t), \quad h_\times^{\perp} = \sin \iota \sin(\Omega_{LT} t). \quad (15)$$

The amplitude ratio of (13) relative to (6) satisfies

$$\frac{h_{LT}}{h} \simeq \frac{1}{8} \left( \frac{\Omega_{LT}}{\Omega_T} \right)^2 \left( \frac{m}{M_T} \right)^{-1} < 1\%. \quad (16)$$

Here, we note that  $M_T/m$  is a ratio of order unity, for  $m$  on the order of a few permille of  $M_H$  in a torus of a few tenths of  $M_\odot$ . Emissions by (13) along the axis are second order in the wobbling angle  $\theta$ ; emissions in directions along the orbital plane are first order in  $\theta$ . This reflects the amplitude of the time-variability of the quadrupole moment in the association projections on the celestial sphere. The associated net luminosity satisfies

$$L_{gw}^{LT} = \frac{1}{10} \Omega_{LT}^6 (\Delta I)^2 \sin^2(2\theta) (1 + 16 \tan^2 \theta), \quad (17)$$

where the first and second part in the bracket correspond to the emissions by  $h^\perp$  and  $h^\parallel$ , respectively at frequencies  $\Omega_{LT}$  and  $2\Omega_{LT}$ . Their contributions are equal when  $\tan \theta = 1/4$  or  $\theta \simeq 14^\circ$ , and the emissions in  $h^\perp$  dominate for wobbling angles  $\theta < 14^\circ$ . In this event, the spectrum produced by (13) varies according to the different dependencies on the viewing angle  $\iota$ . Viewing the source edge on (the observer in the orbital plane) sees  $h_+^\parallel$  at  $2\Omega_{LT}$  and  $h_-^\perp$  at  $\Omega_{LT}$ . Observers along the spin-axis of the central black hole see no lines at  $\Omega_{LT}$  in  $h_+^\perp$  and  $h_-^\perp$ , but a single line at  $2\Omega_{LT}$  in both polarizations of  $h^\parallel$  which are second-order in  $\theta$ .

The lifetime of precession can be calculated by considering the backreaction of gravitational radiation on the torus. Gravitational radiation backreaction forces consist of dynamical self-interactions and radiation-reaction forces [24, 28, 31]. For sources with nonrelativistic motions in the approximation of weak internal gravity, the latter can be modelled by the Burke-Thorne potential as it arises in the  $2\frac{1}{2}$  post-Newtonian approximation. There is no change in the continuity equation in this intermediate order (such as in the second-order post-Newtonian approximation [24, 28, 29]).

Gravitational radiation backreaction on an intrinsic quadrupole moment produced by the  $m = 2$  Papaloizou-Pringle instability, is governed by the small parameter  $\beta = (\pi R \Sigma / 10) (2\Omega_T R)^5$ , or

$$\beta = 5 \times 10^{-5} \left( \frac{\eta}{0.1} \right)^{5/3} \left( \frac{M_T}{0.1 M_\odot} \right) \left( \frac{M_H}{b} \right) \left( \frac{7 M_\odot}{M_H} \right), \quad (18)$$

where  $b \sim M$  denotes the minor radius of the torus in suspended accretion. This backreaction *stimulates* the  $m = 2$  instability [30].

Gravitational radiation backreaction to the precessional motion is due to the low-frequency emissions, since the emissions due to intrinsic multipole mass-moments leave the wobbling angle unaffected. The backreaction of the low-frequency emissions are similar but not identical to that on a freely precessing neutron star. Adapting [25] to the present case, the low-frequency gravitational radiation reaction torque  $T_{gw}^{LT}$  is perpendicular to the symmetry axis of the torus, along which there exists a discrete symmetry in the presence of intrinsic multipole mass-moments. Gravitational radiation reaction torques associated with the latter act along this symmetry axis. It extracts the angular momentum along the spin-axis of the black hole – the axis about which the torus wobbles – and angular momentum associated with the wobble angle itself. In the suspended accretion state, the angular momentum along the spin-axis of the black hole is constant on timescales less than the lifetime of rapid spin of the black hole – the GRB timescale of tens of seconds. We may follow [25], by imposing the constraint

$$J_z = \text{const.} \quad (19)$$

Reduction of the wobbling angle reflects extraction of the kinetic energy of the wobbling motion of the torus. According to [25], we have

$$\dot{\theta} = -\frac{T_{gw}^{LT} \cos \theta}{J}, \quad (20)$$

where  $T_{gw}^{LT} \Omega_{LT} \sin \theta = L_{gw}^{LT}$  [25]. At late times, wobble angle is small, whereby  $\dot{\theta} = -\theta/\tau_\theta$  with  $\tau_\theta^{-1} = (1/10) (\Omega_{LT}/\Omega_H)^4 \Omega_H^4 M_T R^2$  for a torus of mass  $M_T$  and major radius  $R$ . In dimensional units, and using (5), we have

$$\tau_\theta \simeq 2 \times 10^6 \left( \frac{\eta}{0.1} \right)^{-28/3} \left( \frac{M_H}{7 M_\odot} \right) \text{s}. \quad (21)$$

For a wide range of values of  $\eta$  and  $M_H$ , the damping time due to the radiation backreaction is longer than the time-scale of tens of seconds of long GRBs.

We may compare the strength of the high and low-frequency emissions. The luminosity at frequency  $2\Omega_T$  due to an intrinsic quadrupole moment in the torus of mass  $M_T$  satisfies

$$L_{gw}(2\Omega_T) = 3 \times 10^{-4} \left( \frac{\eta}{0.1} \right)^{10/3} \left( \frac{m}{M_T} \right)^2. \quad (22)$$

The luminosity in emissions of  $h^\perp$  associated with a small wobbling angle  $\theta$  satisfies

$$L_{gw}(\Omega_{LT}) = 1.6 \times 10^{-9} \left( \frac{\eta}{0.1} \right)^{10/3} \left( \frac{M_T}{M_H} \right)^2 \theta^2, \quad (23)$$

where we used  $\Omega_{LT}/\Omega_T \simeq \eta$ . In the suspended accretion state, we expect a lumpiness  $m \simeq 0.15\%M_H$  or larger, whereby

$$\frac{m}{M_T} \simeq 10\% \left( \frac{M_H}{7M_\odot} \right) \left( \frac{0.1M_\odot}{M_T} \right). \quad (24)$$

It follows that

$$L_{gw}(\Omega_{LT}) : L_{gw}(2\Omega_T) = 5 \times 10^{-6} \left( \frac{\theta}{0.1} \right)^2 \left( \frac{m}{0.1M_T} \right)^{-2}. \quad (25)$$

We conclude that the low-frequency emissions are low-luminosity lines, relative to the expected emissions from an intrinsic quadrupole moment. This low-luminosity is due to the strong frequency dependence of gravitational radiation, and the limited source of (kinetic) energy in the wobbling motion of the torus. In contrast, the gravitational radiation produced by the intrinsic quadrupole moment is produced by the Keplerian frequency and it derives from the large reservoir in spin-energy of the the black hole.

To summarize, we predict long bursts of gravitational radiation in GRB-supernovae from rotating black holes with an energy output of  $4 \times 10^{53}$  erg. These bursts represent the catalytic conversion of black hole-spin energy into gamma-rays and gravitational waves, and an accompanying supernova. The gravitational wave-luminosity is dominated by emissions from multipole mass-moments in the torus for sufficiently slender tori (minor-to-major radius of the torus less than about 0.75 for a mass-moment  $m = 1$ , and less than about 0.33 for mass-moment  $m = 2$  [13]). The duration is set by the lifetime of rapid spin of the black hole. A misalignment of the torus and the black hole conceivably represents a remnant feature of the angular momentum distribution in the progenitor binary of a massive star and its companion. It produces phase-modulation in the observed strain-amplitude.

A detailed understanding of the wave-form of gravitational radiation is important in applying matched filtering. For matched filtering, we have the signal-to-noise estimate [13]

$$\left( \frac{S}{N} \right)_{mf} \simeq 8 \left( \frac{S_h^{1/2}(500\text{Hz})}{4 \times 10^{-24}\text{Hz}^{-1/2}} \right)^{-1} \eta_{0.1}^{-3/2} M_7^{5/2} d_8^{-1}. \quad (26)$$

Presently, the second science run in LIGO shows  $S_h^{1/2}(500\text{Hz}) = 5 \times 10^{-22}\text{Hz}^{-1/2}$  [1], which corresponds to  $S/N = 8$  for a source at  $d_L \simeq 1\text{Mpc}$ . This will improve to a sensitivity range of 100Mpc with the Advanced LIGO noise-strain amplitude target  $S_h^{1/2}(500\text{Hz}) = 4 \times 10^{-24}\text{Hz}^{-1/2}$ . This sensitivity range corresponds a true event-rate (seen in gravitational radiation; seen and unseen in GRB-emissions) of one per year, based on a GRB beaming factor of about 500 [11, 12].

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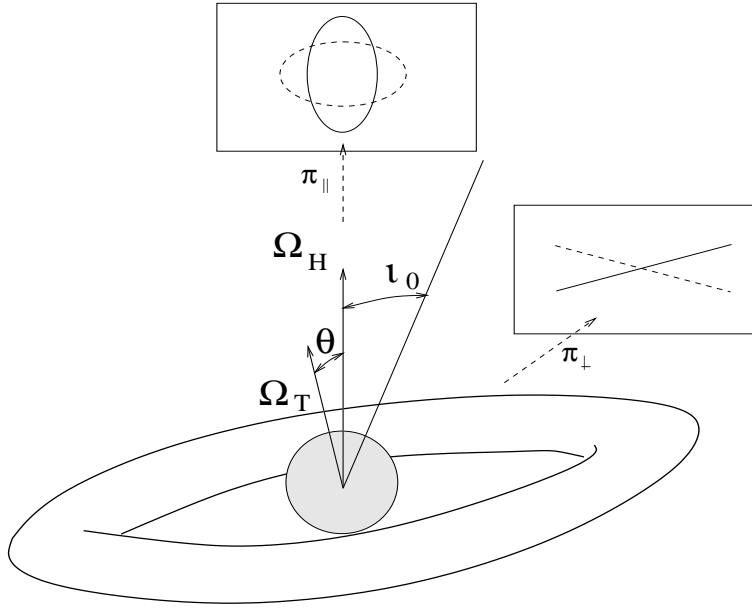


FIG. 1: The geometry of Lense-Thirring precessing torus with angular velocity  $\Omega_T$  around a rotating black hole with angular velocity  $\Omega_H$ . The wobbling angle is  $\theta$ . The line-of-sight has an angle  $\iota_0$  relative to the spin-axis of the black hole. The angle of the orientation of the torus to the line-of-sight varies in time, between  $\iota_0 - \theta$  and  $\iota_0 + \theta$ . This introduces phase-modulation in the observed strain-amplitude of emissions by an intrinsic quadrupole moment in the torus. Low-frequency spectral anisotropy is produced by apparent quadrupole moments, corresponding to projections  $\pi_{||}$  and  $\pi_{\perp}$  of the torus onto the celestial sphere in directions along and orthogonal to the spin-axis of the black hole. The associated modulations are at twice and, respectively, once the Lense-Thirring precession frequency.

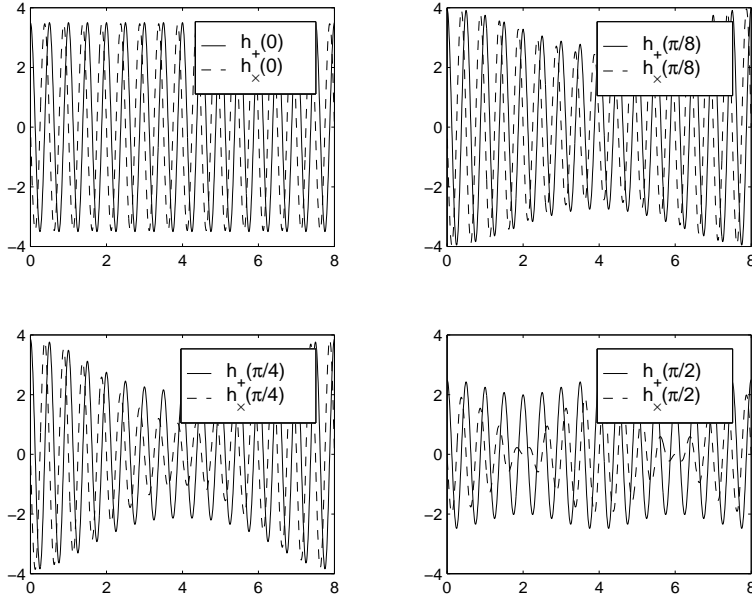


FIG. 2: The strain-amplitudes  $h_+$  and  $h_{\times}$  around twice the Keplerian angular frequency  $\Omega_T$  are produced by a quadrupole mass-moment in the torus in a state of suspended accretion. The observed strain-amplitudes are subject to phase-modulation by Lense-Thirring precession of the torus. For wobbling angles which are small relative to the mean angle to the line-of-sight, the resulting amplitude modulation produces two side-bands to the carrier frequency  $2\Omega_T$ , separated by the Lense-Thirring frequency  $\Omega_{LT}$ . Shown are the strain-amplitudes for the case of  $\iota_0 = 0, \pi/8, \pi/4, \pi/2$  for a wobbling angle of  $30^\circ$  and a Lense-Thirring precession frequency of  $1/8$  of the Keplerian frequency. The amplitude corresponds to a source at unit distance, and the index refers to the number of Keplerian periods. Additionally, small contributions to the strain amplitude arise at once and twice  $\Omega_{LT}$  in response to apparent quadrupole moments in the projections of the torus onto the celestial sphere.